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# Determination Of Visibility Time Of Leo And Meo Satellites With Circular Orbits

**Kufre M. Udofia**
Department of Electrical/Electronic and Computer Engineering, University of Uyo, Nigeria
kmudofiaa@uniuyo.edu.ng

*Abstract*— **In this paper, determination of the visibility time of Low Earth Orbit (LEO) satellite and Medium Earth Orbit (MEO) satellite with circular orbits is presented. The study presented relevant mathematical expressions for computing the visibility time of the satellites in two different scenarios, namely, the visibility time without restriction on the minimal zenithal (θ) angle, as well as the case where there is restriction on the minimal zenithal angle. Sample LEO (Iridium) satellite with altitude of 780 km and MEO satellite with altitude of 20,000 km were used for numerical examples. The results of the visibility computation for the Iridium satellite for the case of no restriction on the minimal zenithal angle (that is, with θ = 0°) is 903.96 seconds. The case where there is restriction on the minimal zenithal angle, with θ = 5°, the visibility time is 750.76 seconds. Also, with θ = 15°, the visibility time is 522.62 seconds. The visibility time of a MEO satellite with θ = 0° is 18003.66 seconds, with θ = 5° the visibility time is 16832.20 seconds, and with θ = 15° the visibility time is 14565.77 seconds. Simple exponential expressions relating the visibility time to θ for the LEO and MEO satellites were derived from the results. In all, the MEO satellite has higher visibility time than the LEO satellite. Also, the higher the restriction on the minimal zenithal angle, the lower the visibility time of the satellite.**

***Keywords— LEO satellite, visibility time , Zenithal angle, Iridium satellite, Circular Orbits , MEO satellite***

## 1. Introduction

Over the years, satellite technologies have been developed and deployed for diverse applications across the globe [1,2,3]. The suitability of a satellite for a given application depends on certain parameters pertaining to the satellite. Accordingly, today, there are different kinds of satellites classified based on different criteria. One of the criteria for classifying satellites is based on the height of the satellite orbit. In this wise, there are Low Earth Orbit (LEO) satellite, Medium Earth Orbit (MEO) satellite, Geo-synchronous (GEO) satellite and High Elevation Orbit (HEO) satellite [4,5,6,7,8,9,10,11,12,13,14,16,17]. The orbital height affect the visibility of the satellite from a given earth station location as well as the elevation angle of the satellite–earth station link. Furthermore, the orbital path can assume different shapes, namely, circular, elliptical, near circular, highly elliptical, parabolic and hyperbolic paths. In this paper, the focus is on the visibility of LEO and MEO satellites with circular orbit.

Generally, satellites communication is a wireless communication which can exist between earth station and the satellite or a satellite with other satellite [18,19,20]. Accordingly, like other wireless communication systems, the ability to receive signal from a satellite depends on a number of factors like the propagation loss, the transmitter power, the communication path length, among others [21,22,23,24,25,26,27,28,29,30]. In addition, the motion of satellite also affect satellite visibility. Notably, the visibility of satellite indicates the ability of receiver to detect or sense or receive signal from the satellite due to the relative position of the satellite with respect to the receiver. Particularly, different orbital shapes affect the percentage of time the satellite can be visible from a given earth station. Also, the range of applicable elevation angle for visibility can also determine the percentage of time the satellite can be visible. Accordingly, the study examined the visibility time of the satellites in two different scenarios, namely, the visibility time without restriction on the minimal zenithal angle, as well as the case where there is restriction on the minimal zenithal angle. Requisite analytical models for the computation of the visibility of the satellite are presented. Also, sample satellite parameters are used for numerical examples.

## 2. Methodology

The study considered the visibility time for satellite on circular orbit in two different scenarios, namely, first case with no restriction on the minimal zenithal angle and second case with restriction on the minimal zenithal angle.

### 2.1 The visibility arc diagram for a circular orbit with no restriction on the minimal zenithal angle

The satellite visibility diagram for a circular orbit with no restriction on the minimal zenithal angle is shown in Figure 1. In this case, the satellite is visible to a point, P on the earth surface, as long as the satellite is located at any point above the local horizon of the point P. This means, as long as the satellite is within the orbital arc XQW.

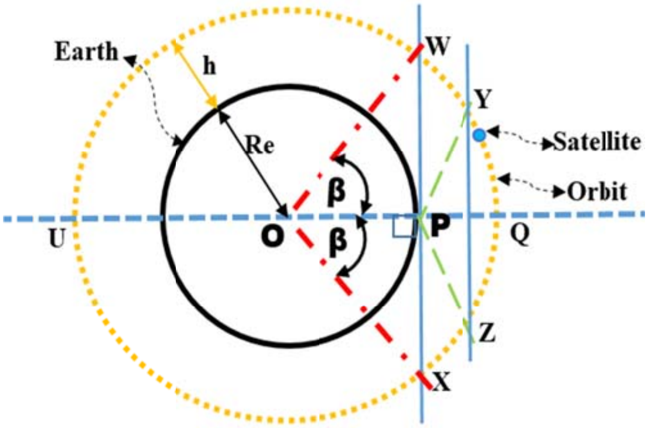

Figure 1 The satellite visibility diagram for a circular orbit with no restriction on the minimal zenithal angle

In order to determine the visibility time, we consider the earth with radius $R_e$ orbited by a satellite along an orbit with altitude, h which gives an orbital radius, $R_s$ , where;

$$R_s = R_e + \text{h} \tag{1}$$

At a point, P on the earth surface, and with no restriction on the minimal zenithal angle, the satellite is visible as long as it is within the arc XQW. The angle subtended at the centre, O by arc XQW is 2β, where β is angle POX. Now,

$$\text{Cos}(\beta) = \frac{OP}{OX} = \frac{R_e}{R_e+h} = \frac{R_e}{R_s} = \frac{1}{\eta} \tag{2}$$

The satellite's orbital period is take to be equal to the Keplerian period, $T_o$ where

$$T_o = 2\pi\sqrt{\frac{(R_e+h)^3}{\mu}} = 2\pi\sqrt{\frac{(R_s)^3}{\mu}} \tag{3}$$

Where $\mu = 398600\ Km^3/s^2$.

The visibility time, denoted as $\Delta t_v$ is given as the time it takes the satellite to move along arc XQW. Hence, the following relationships apply;

$$\frac{\Delta t_v}{T_o} = \frac{2\beta}{2\pi} \tag{4}$$

Therefore,

$$\Delta t_v = \left(\frac{\beta}{\pi}\right) T_o \tag{5}$$

$$\Delta t_v = \left(2\sqrt{\frac{(R_s)^3}{\mu}}\right) Cos^{-1}\left(\frac{R_e}{R_s}\right) \tag{6}$$

### 2.2 The visibility arc diagram for a circular orbit with restriction on the minimal zenithal angle, φ

In the description so far, there is no restriction on the minimal zenithal angle of sight which in this case is 90°, that is, angle QPX in Figure 1. If a restriction is imposed on MZAS, denoted as angle φ, such that the satellite is required to be above a certain angle (θ) above the horizon, (shown in Figure 2 ) where MZAS, denoted as angle φ is given as ;

$$\varphi = 90 - \theta \tag{7}$$

In this case, $\text{Cos}(\beta) \neq \frac{1}{\eta}$ , rather ,

$$\text{Cos}(\beta) - \left(\frac{1}{\text{Tan}(\varphi)}\right) \text{Sin}(\beta) = \frac{1}{\eta} \tag{8}$$

Now, let

$$\text{H} = \frac{1}{\eta} \tag{9}$$

$$\text{Z} = \frac{1}{\text{Tan}(\varphi)} \tag{10}$$

Then,

$$\beta = 2\left(\text{Tan}^{-1}\left(\frac{\sqrt{(1+Z^2-H^2)}-Z}{1+\text{H}}\right)\right) \tag{11}$$

Again, based on the value of β in Eq 11, the visibility time, $\Delta t_v$ is given as,

$$\Delta t_v = \left(\frac{\beta}{\pi}\right) T_o \tag{12}$$

Where $T_o$ is as defined in Eq 3.

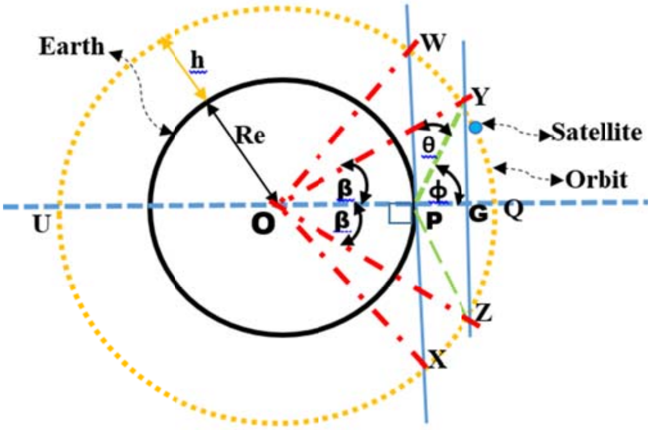

Figure 2 The satellite visibility diagram for a circular orbit with restriction on the minimal zenithal angle, φ

## 3. Results and Discussions

The visibility of Iridium satellite which is a LEO satellite with circular orbit at an altitude of 780 Km is considered with Earth radius of 6,378.14 km. The results of the visibility computation for the Iridium satellite for the case of no restriction on the minimal zenithal angle (that is, with θ = 0°) are shown in Table 1. In this case, the visibility time of the Iridium satellite is 903.96 seconds, which is equivalent to 15.07 minutes or 0.25 hours. The results of the visibility computation for the Iridium satellite for the case where there is restriction on the minimal zenithal angle, with θ = 5° are shown in Table 2 while that with θ = 15° are shown in Table 3. The results showed that with θ = 5°, the visibility time of the Iridium satellite is 750.76 seconds, which is equivalent to 12.51 minutes or 0.21 hours. Similarly, with θ = 15°, the visibility time of the Iridium satellite is 522.62 seconds, which is equivalent to 8.71 minutes or 0.15 hours. The graph of the visibility time, Δtv (s) versus the minimum angle above local horizon, θ (°) for the LEO Iridium satellite is shown in Figure 3. The analytical expression relating Δtv (s) to θ for the LEO Iridium satellite is given in Eq 13.

Table 1 The results of the visibility computation for the Iridium satellite for the case of no restriction on the minimal zenithal angle (that is, with θ = 0°)

| Altitude, h (Km) | Orbital period,To (s) | Minimum angle above local horizon, θ (°) | Zenithal angle, ϕ | Angle β (rad) | Angle β (deg) | Visibility time, Δtv (s) | Visibility time, Δtv (min) | Visibility time, Δtv (hour) |
|---|---|---|---|---|---|---|---|---|
| 780 | 6027.1 | 0.0 | 90 | 0.47118 | 27.00 | 903.96 | 15.07 | 0.25 |

Table 2 The results of the visibility computation for the Iridium satellite for the case where there is restriction on the minimal zenithal angle (with θ = 5°)

| Altitude, h (Km) | Orbital period,To (s) | Minimum angle above local horizon, θ (°) | Zenithal angle, ϕ | Angle β (rad) | Angle β (deg) | Visibility time, Δtv (s) | Visibility time, Δtv (min) | Visibility time, Δtv (hour) |
|---|---|---|---|---|---|---|---|---|
| 780 | 6027.1 | 5.0 | 85 | 0.391329 | 22.42 | 750.76 | 12.51 | 0.21 |

Table 3 The results of the visibility computation for the Iridium satellite for the case where there is restriction on the minimal zenithal angle (with θ = 15°)

| Altitude, h (Km) | Orbital period,To (s) | Minimum angle above local horizon, θ (°) | Zenithal angle, ϕ | Angle β (rad) | Angle β (deg) | Visibility time, Δtv (s) | Visibility time, Δtv (min) | Visibility time, Δtv (hour) |
|---|---|---|---|---|---|---|---|---|
| 780 | 6027.1 | 15.0 | 75 | 0.272409 | 15.61 | 522.62 | 8.71 | 0.15 |

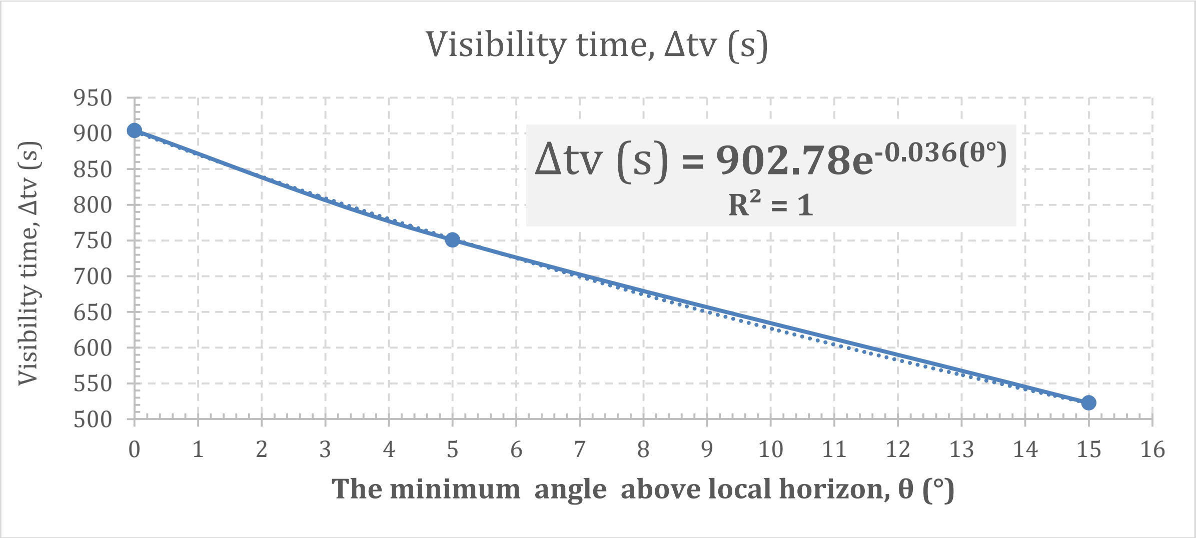

Figure 3 The plot of the visibility time, Δtv (s) versus the minimum angle above local horizon, θ (°) for the LEO Iridium satellite

$$\Delta tv\ (s) = \mathbf{902.78e^{-0.036(\theta°)}} \quad (13)$$

Apart from the LEO satellite, the visibility of a MEO satellite with circular orbit at an altitude of 20,000 Km is also considered. The results of the visibility computation for the MEO satellite for the case of no restriction on the minimal zenithal angle (that is, with θ = 0°) are shown in Table 4. In this case, the visibility time of the MEO satellite is 18003.66 seconds, which is equivalent to 300.06 minutes or 5 hours. The results of the visibility computation for the MEO satellite for the case where there is restriction on the minimal zenithal angle, with θ = 5° are shown in Table 5 while that with θ = 15° are shown in Table 6. The results showed that with θ = 5° , the visibility time of the MEO satellite is 16832.20 seconds, which is equivalent to 280.54 minutes or 4.68 hours. Similarly, with θ = 15°, the visibility time of the MEO satellite is 14565.77 seconds, which is equivalent to 242.76 minutes or 4.05 hours. The graph of the visibility time, Δtv (s) versus the minimum angle above local horizon, θ (°) for the MEO satellite is shown in Figure 4. The analytical expression relating Δtv (s) to θ for the MEO satellite is given in Eq 14.

**Table 4** The results of the visibility computation for the MEO satellite for the case of no restriction on the minimal zenithal angle (that is, with θ = 0°)

| Altitude, h (Km) | Orbital period, To (s) | Minimum angle above local horizon, θ (°) | Zenithal angle, ϕ | Angle β (rad) | Angle β (deg) | Visibility time, Δtv (s) | Visibility time, Δtv (min) | Visibility time, Δtv (hour) |
|---|---|---|---|---|---|---|---|---|
| 20000 | 42636.1 | 0.0 | 90 | 1.32658 | 76.01 | 18003.66 | 300.06 | 5.00 |

**Table 5** The results of the visibility computation for the MEO satellite for the case where there is restriction on the minimal zenithal angle, with θ = 5°

| Altitude, h (Km) | Orbital period, To (s) | Minimum angle above local horizon, θ (°) | Zenithal angle, ϕ | Angle β (rad) | Angle β (deg) | Visibility time, Δtv (s) | Visibility time, Δtv (min) | Visibility time, Δtv (hour) |
|---|---|---|---|---|---|---|---|---|
| 20000 | 42636.1 | 5.0 | 85 | 1.240261 | 71.06 | 16832.20 | 280.54 | 4.68 |

**Table 6** The results of the visibility computation for the MEO satellite for the case where there is restriction on the minimal zenithal angle, with θ = 15°

| Altitude, h (Km) | Orbital period, To (s) | Minimum angle above local horizon, θ (°) | Zenithal angle, ϕ | Angle β (rad) | Angle β (deg) | Visibility time, Δtv (s) | Visibility time, Δtv (min) | Visibility time, Δtv (hour) |
|---|---|---|---|---|---|---|---|---|
| 20000 | 42636.1 | 15.0 | 75 | 1.073262 | 61.49 | 14565.77 | 242.76 | 4.05 |

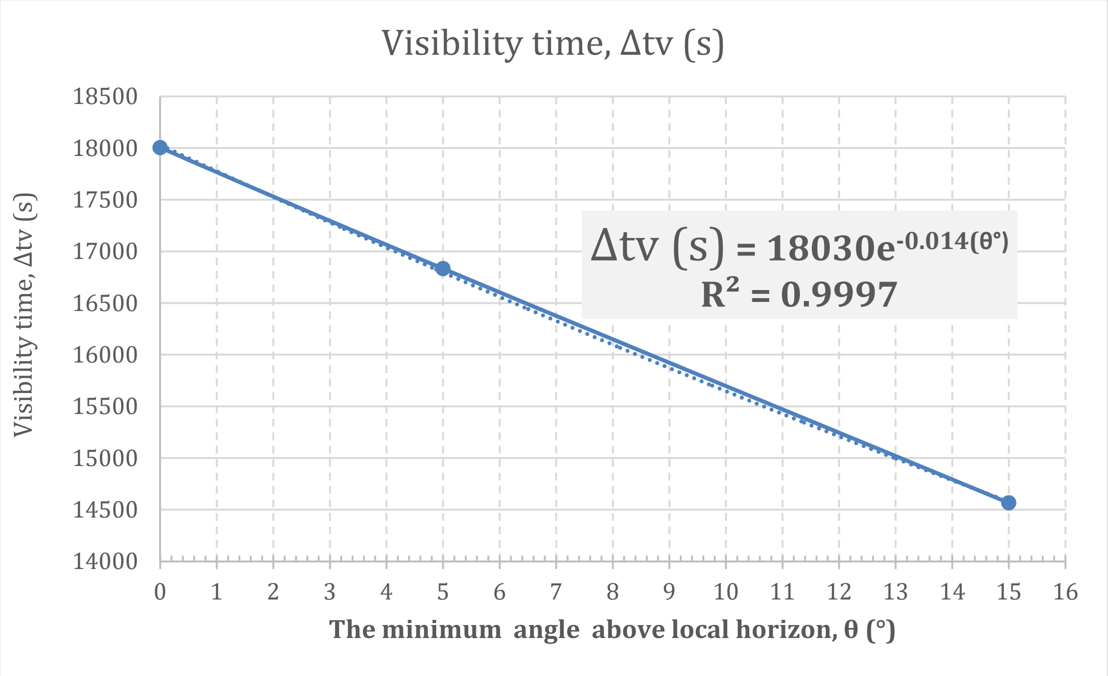

Figure 4 The plot of the visibility time, Δtv (s) versus the minimum angle above local horizon, θ (°) for the MEO satellite

$$\Delta tv\ (s) = 18030e^{-0.014(\theta°)} \qquad (14)$$

## 4. Conclusion

Computation of the visibility time of Low earth orbit (LEO) satellite and Medium Earth Orbit (MEO) satellite is presented. The study considered the visibility of the satellites without restriction on the minimal zenithal angle, as well as the case where there is restriction on the minimal zenithal angle. Sample LEO and MEO satellites were used for numerical examples. The results showed that the MEO satellite has higher visibility time than the LEO satellite. Also, the higher the restriction on the minimal zenithal angle, the lower the visibility time of the satellite.

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