Design Of Modular Program For Evaluation Of Visibility Time Of Satellite With Highly Eccentric Orbit

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***Abstract*— The design of modular program for parametric analysis of visibility time of satellite with highly eccentric orbit was presented. The highly eccentric orbits elliptical and their visibility time is significantly influenced by the value of their eccentricity. In this paper, the mathematical expression for parametric analysis of the visibility time of such elliptical orbits are presented along with the pseudocode design for a modular program that can be used to carry out the parametric analysis. The program was implemented using the Visual Basic program for applications. The implementation of the program was performed on Microsoft Excel application under Windows operating system. The orbital parameters of five case study highly eccentric orbit satellites were used for the numerical examples. The range of values of eccentricity values of the selected satellites is 0.596 to 0.889 while the range of values of the semi major axis values of the selected satellites is 21136 km to 97697 km. The results show that one, both visibility time and percentage of visibility time per orbital period increase with increase in eccentricity, and both visibility time and percentage of visibility time per orbital period decreases with increase in minimum elevation angle restriction. In any case, the orbital period is not affected by the eccentricity. Also, both visibility time and percentage of visibility time per orbital period decrease with increase in minimum elevation angle restriction, and both orbital Period and percentage of visibility time per orbital period increase with increase in eccentricity. In any case, the orbital period is not affected by the minimum elevation angle. Furthermore, the orbital period, visibility time and percentage of visibility time per orbital period increase with increase in orbital semi-major axis.**

Keywords*— *Modular Program, Satellite Visibility Time, Highly Eccentric Orbit, Eccentricity, Orbital Period

1. Introduction

Satellites orbiting the earth are usually deployed to be accessed on earth for various purposes [1,2,3,4,5,6,7,8,9,10,11,12,13]. The accessibility of the satellite depends on its orbital location, the altitude of its orbit above the earth and the specific location on earth from where the satellite is to be accessed [14,15,16,17,18,19,20,21]. Also, the satellites complete full circle orbital motion in a time frame called orbital period [22,23,24,25,26,27,28,29,30,31]. Within the orbital period, a satellite may be visible to some locations on earth for a fraction of their orbital period. The maximum time the satellite is visible to earth location is termed visibility time [32,33,34,35,36,37,38,39]. Within the visibility time, a satellite can communicate with earth stations, images of locations on the earth came be captured by the satellite-born cameras and other satellite based applications can be implemented.

In any case, the visibility time for elliptical orbits is significantly affected by the eccentricity of the orbit. Orbits with eccentricity that is approximately zero are considered as circular orbit whereas those orbits with eccentricity values as high as 0.5 and above are elliptical in shape. The visibility time of such elliptical or highly eccentric orbit is dependent on the eccentricity and orbital period. The orbital period however, is proportional to the orbital altitude. So, with orbital period or altitude and eccentricity, the visibility time of highly eccentric orbits can be determined.

In this paper, the focus is development of modular program for automated computation of the visibility time of elliptical orbits with relatively high eccentricity values. The program used different functions or modules to compute the satellite's visibility time based on the specific combination of available input dataset. The program also considered the minimum elevation angle restriction as part of the input parameters. In all, the program enables parametric analysis of the effect of changes in any of the given input parameters on the satellite's visibility time. Some satellites that are currently in highly eccentric orbits are used for numerical examples.

2. Methodology

The works by [the authors in 40] on the visibility time of highly eccentric orbit is a procedural program that failed to include the capability for automated parametric analysis. The work provided algorithm for one time computation of the visibility time. In that case, performing parametric analysis requires multiple explicit rerun or the program code. In order to accommodate parametric analysis of the visibility time as function of selected orbital parameters modular approach to program design is implemented.

Notably, the visibility time, Δt_{vheS} of highly eccentric orbit is given by [40] in respect of orbital semi-major axis (a) in km and the orbital eccentricity (e) along with the constant parameter, μ, $\left(\text{where } \mu = 398600 \frac{Km^3}{s^2}\right)$. Specifically, the visibility time, Δt_{vheS} in seconds is defined as;

$$\Delta t_{vheS} = \left(1 - \frac{\left(2\left(\tan^{-1}\left(\sqrt{\frac{(1-e)}{(1+e)}}\right)\right) - \left(e\left(\sqrt{(1-e^2)}\right)\right)\right)}{\pi}\right)\left(2\pi\sqrt{\frac{a^3}{\mu}}\right) \quad (1)$$

Where $\left(2\pi\sqrt{\frac{a^3}{\mu}}\right)$ is the orbit period, T_o of the satellite. That is,

$$T_o = 2\pi\sqrt{\frac{a^3}{\mu}} \quad (2)$$

If T_o is given, then, the orbital semi-major axis (a) can be computed as;

$$a = \sqrt[3]{\left(\mu\left(\frac{T_o}{2\pi}\right)^2\right)} \quad (3)$$

When restriction of ε radians is imposed as the minimum elevation angle, then, the visibility angle becomes;

$$\Delta t_{vheS} = \left(1 - \frac{\left(2\left(\tan^{-1}\left(\sqrt{\frac{(1-e)}{(1+e)}}\right)\right) - \left(e\left(\sqrt{(1-e^2)}\right)\right)\right)}{\pi}\right)\left(2\pi\sqrt{\frac{a^3}{\mu}}\right)\left(1 - \frac{2}{\pi}(\varepsilon)\right) \quad (4)$$

$$\Delta t_{vheS} = \left(1 - \frac{\left(2\left(\tan^{-1}\left(\sqrt{\frac{(1-e)}{(1+e)}}\right)\right) - \left(e\left(\sqrt{(1-e^2)}\right)\right)\right)}{\pi}\right)\left(2\pi\sqrt{\frac{a^3}{\mu}}\right)(f_\varepsilon) \quad (5)$$

$$\Delta t_{vheS} = \left(1 - \frac{\left(2\left(\tan^{-1}\left(\sqrt{\frac{(1-e)}{(1+e)}}\right)\right) - \left(e\left(\sqrt{(1-e^2)}\right)\right)\right)}{\pi}\right)(T_o)(f_\varepsilon) \quad (6)$$

Where

$$(f_\varepsilon) = 1 - \frac{2}{\pi}(\varepsilon) \quad (7)$$

Therefore, from equation 5 and equation 6, the parametric analysis can be conducted with respect to the following four parameters, e, ε, a and T_o.In the modular program design adopted in this paper, the parametric analysis with respect to each of the four parameters is designed as a separate module which will be called from the main module. Also, each of the four modules compute the visibility times and formats the output according to a tabular format. The choice of which parametric analysis is to be performed is determined by the user and the main module provides the user with the opportunity to make the choice and then call the appropriate module to carry out the parametric analysis based on the specific user's choice.

The five modules in this program are;

i. Module Main

   The Module Main is the first module to run when the program is implemented and it is the one that will end the program when the user exit the program of completes all the desired parametric analysis. It provides the user with the four options to choose the specific parametric analysis that is desired. The four options corresponds to the four module, from Module_1 to Module_4. The option is presented as a numeric data that consists of 1, 2, 3, and 4 and the value corresponds with numeric values on the four modules, namely, '1' for Module_1 , '2' for Module_2, '3' for Module_3 and '4' for Module_4.

ii. Module_1 VisTim_by_ec

   This module is used to compute the visibility time for different values of eccentricity, e. In this case, other parameters are kept constant while the eccentricity is varied between two values, emn and emx where emn ≤ e ≤ emx and e is the specified orbital eccentricity of the orbit.

   On the other hand, the module can be used to compute the visibility time of different satellites based on their eccentricities. In this case, the number of satellites that will be considered is indicated as Xec and then Module_1 VisTim_by_ec will input the Xec different eccentricity values and compute the visibility time for each of them.

   Essentially, there are two versions of Module_1 VisTim_by_ec, in the first version, a range of values is selected for the eccentricity and the actual values of eccentricity used for the computation are automatically chosen from the given range. On the other hand, in the second version, the individual Xec eccentricity values are manually read and stored and then used to compute the visibility time one by one until each of the Xec eccentricity values has been used.

iii. Module_2 VisTim_by_εl
This module is used to compute the visibility time for different values of minimum elevation angle restriction, ε. In this case, other parameters are kept constant while the ε is varied between two values; 0 and εmx where εmx $\leq \frac{\pi}{2}$.

iv. Module_3 VisTim_by_ax
This module is used to compute the visibility time for different values of orbital semi-major axis, a. In this case, other parameters are kept constant while the orbital semi-major axis (a) is varied between two values, amn and amx where amn ≤ a ≤ amx and a is the specified orbital semi-major axis (a) of the satellite.
In addition, the module can be used to compute the visibility time of different satellites based on their semi-major axis, a. In this case, the number of satellites that will be considered is indicated as Xax and then Module_2 VisTim_by_ax will input the Xax different semi-major axis values and compute the visibility time for each of them.
Essentially, there are two versions of Module_2 VisTim_by_ax, in the first version, a range of values is selected for the semi-major axis and the actual values of semi-major axis used for the computation are automatically chosen from the given range. On the other hand, in the second version, the individual Xax semi-major axis values are manually read and stored and then used to compute the visibility time one by one until each of the Xax semi-major axis values has been used.

v. Module_4 VisTim_by_To
This module is used to compute the visibility time for different values of orbital period, To. In this case, other parameters are kept constant while the orbital period, To is varied between two values, Tomn and Tomx where Tomn ≤ To ≤ Tomx and To is the specified orbital period, To of the satellite.
Furthermore, the module can be used to compute the visibility time of different satellites based on their orbital period, To. In this case, the number of satellites that will be considered is indicated as Xto and then Module_4 VisTim_by_To will input the Xto different orbital period values and compute the visibility time for each of them.
In essence, there are two versions of Module_4 VisTim_by_To, in the first version, a range of values is selected for the orbital period and the actual values of orbital period used for the computation are automatically chosen from the given range. On the other hand, in the second version, the individual Xto orbital period values are manually read and stored and then used to compute the visibility time one by one until each of the Xto orbital period values has been used.

The detailed modules

Module main ()

```
    Define μ = 398600  // in km³/s²
    Input ParCat /* Choose the parametric analysis category:
                    '1' for visibility time versus eccentricity (e),
                    '2' for visibility time versus minimum elevation angle restriction (ε),
                    '3' for visibility time versus orbital semi-major axis (a), and
                    '4' for visibility time versus orbital period (To)
                    */
    If ParCat = 1 Then
        Call Module_1 VisTim_by_ec
    ElseIf ParCat = 2 Then
        Call Module_2 VisTim_by_εl
    ElseIf ParCat = 3 Then
        Call Module_3 VisTim_by_ax
    Else
        Module_4 VisTim_by_To
    EndIf
    Return
End Module main
```

```
Module_1 VisTim_by_ec ()
    Input InpCatEc /* Choose the input category:
                    '1' for range of values-eccentricity
                    '2' for explicit eccentricity input
                    */
    If InpCatEc = 1 Then
        Call Module_1 VisTim_by_ec_byRange
    Else
        Call Module_2 VisTim_by_ec_byExpInp
    EndIf
    Return
End Module_1 VisTim_by_ec ()
```



```
Module_1 VisTim_by_ec ()
    Input InpCatEc /* Choose the input category:
                    '1' for range of values of eccentricity, e
                    '2' for explicit eccentricity input
                    */
    If InpCatEc = 1 Then
        Call Module_1 VisTim_by_ec_byRange
    Else
        Call Module_1 VisTim_by_ec_byExpInp
    EndIf

    Return
End Module_1 VisTim_by_ec ()
```


Module_1 VisTim_by_ec_byRange ()

Input emn //input lowest value of eccentricity
Input emx //input highest value of eccentricity
Input Dmx //input number of subdivisions of the range emn to emx
Declare the array **Ec**[Dmx] //array **Ec**[Dmx] stores the 1,2,3,...Dmx eccentricity values
Declare the array VtHeS[Dmx] //array VtHeS [Dmx] holds the 1,2,3,... Dmx visibility times in seconds
Declare the array VtHeM[Dmx] //array VtHeM [Dmx] holds the 1,2,3,... Dmx visibility times in minutes
Declare the array VtHeH[Dmx] //array VtHeH [Dmx] holds the 1,2,3,... Dmx visibility times in HOUR
Input a //input orbital semi-major axis in km
Input ε //input minimum elevation angle restriction value in radian
For M = 1 , M ≤ Dmx, M = M + 1

$$Ec[\mathrm{M}] = M\left(\frac{\mathrm{emx}-\mathrm{emn}}{\mathrm{Dmx}}\right)$$

VtHeS[M] = compute_VT (**Ec**[M], a, ε) // Call the Module that computes the Mth visibility time when e , a and ε are given and the result is in seconds)

$$\mathrm{VtHeM[M]} = \left(\frac{\mathrm{VtHeS[M]}}{60}\right)$$

$$\mathrm{VtHeH[M]} = \frac{\mathrm{VtHeS[M]}}{3600}$$

Output M, **Ec**[M], a, ε , VtHeS[M], VtHeM[M], VtHeH[M]
Next For Loop
End For Loop

End Module_1 VisTim_by_ec_byRange ()

Module_1 VisTim_by_ec_byExpInp()

```
Input Dmx // Dmx is the number of different eccentricities that the user want to consider
Declare the array Ec[Dmx] // array Ec[Dmx] stores the 1,2,3,...Dmx eccentricity values
Declare the array VtHeS[Dmx] // array VtHeS [Dmx] holds the 1,2,3,... Dmx visibility times in seconds
Declare the array VtHeM[Dmx] // array VtHeM [Dmx] holds the 1,2,3,... Dmx visibility times in minutes
Declare the array VtHeH[Dmx] // array VtHeH [Dmx] holds the 1,2,3,... Dmx visibility times in HOUR
Input a // input orbital semi-major axis in km
Input ε // input minimum elevation angle restriction value in radian
For M = 1 , M ≤ Dmx, M = M + 1
    Input Ec[M] // Input the Mth eccentricity
    VtHeS[M] = compute_VT (Ec[M], a, ε) // Call the Module that computes the Mth visibility time when e , a and ε are given and the
    result is in seconds)
```

$$VtHeM[M] = \left(\frac{VtHeS[M]}{60}\right)$$

$$VtHeH[M] = \frac{VtHeS[M]}{3600}$$

```
    Output M, Ec[M], a, ε , VtHeS[M], VtHeM[M], VtHeH[M]
Next For Loop
End For Loop
Return
```

End Module_1 VisTim_by_ec_byExpInp()

Module_2 VisTim_by_εl ()

```
Input InpCatEl /* Choose the input category:
        '1' for range of values of minimum elevation angle restriction, ε (in radian)
        '2' for explicit minimum elevation angle restriction, ε input
        */
If InpCatEl = 1 Then
    Call Module_2 VisTim_by_εl_byRange()
Else
    Call Module_2 VisTim_by_εl_byExpInp()
EndIf

Return
```

End Module_2 VisTim_by_εl ()

Module_2 VisTim_by_εl_byRange ()

```
Input elmn // input lowest value of ε in radian
Input elmx // input highest value of ε in radian
Input Dmx // input number of subdivisions of the range elmn to elmx
Declare the array El[Dmx] // array El[Dmx] stores the 1,2,3,...Dmx eccentricity values
Declare the array VtHeS[Dmx] // array VtHeS [Dmx] holds the 1,2,3,... Dmx visibility times in seconds
Declare the array VtHeM[Dmx] // array VtHeM [Dmx] holds the 1,2,3,... Dmx visibility times in minutes
Declare the array VtHeH[Dmx] // array VtHeH [Dmx] holds the 1,2,3,... Dmx visibility times in HOUR
Input a // input orbital semi-major axis in km
Input e // input eccentricity
For M = 1 , M ≤ Dmx, M = M + 1
```

$$El[M] = M\left(\frac{elmx - elmn}{Dmx}\right)$$

```
VtHeS[M] = compute_VT (e, a, El[M]) // Call the Module that computes the Mth visibility time when e , a and ε are given and the result is in
    seconds)
```

VtHeM[M] $= \left(\frac{\text{VtHeS[M]}}{60}\right)$

VtHeH[M] $= \frac{\text{VtHeS[M]}}{3600}$

Output M, e, a, **Ec**[M] , VtHeS[M], VtHeM[M], VtHeH[M]

Next For Loop

End For Loop

End Module_2 VisTim_by_εl_byRange ()

Module_2 VisTim_by_εl_byExpInp()

Input Dmx // Dmx is the number of different minimum elevation angle restriction, ε (in radian) that the user want to consider

Declare the array **Ec**[Dmx] // array **Ec**[Dmx] stores the 1,2,3,...Dmx eccentricity values

Declare the array VtHeS[Dmx] // array VtHeS [Dmx] holds the 1,2,3,... Dmx visibility times in seconds

Declare the array VtHeM[Dmx] // array VtHeM [Dmx] holds the 1,2,3,... Dmx visibility times in minutes

Declare the array VtHeH[Dmx] // array VtHeH [Dmx] holds the 1,2,3,... Dmx visibility times in HOUR

Input a // input orbital semi-major axis in km

Input e // input eccentricity

For M $= 1$, M $\leq$ Dmx, $M =$ M + 1

Input El[M] // Input the Mth minimum elevation angle restriction, ε (in radian)

VtHeS[M] = compute_VT (e,a, **El**[M],) // Call the Module that computes the Mth visibility time when **e** , **a** and ε are given and the result is in seconds)

VtHeM[M] $= \left(\frac{\text{VtHeS[M]}}{60}\right)$

VtHeH[M] $= \frac{\text{VtHeS[M]}}{3600}$

Output M, **e**, a, **El**[M], VtHeS[M], VtHeM[M], VtHeH[M]

Next For Loop

End For Loop

Return

End Module_2 VisTim_by_εl_byExpInp()

Module_3 VisTim_by_ax ()

Input InpCatAx /* Choose the input category:
‘1’ for range of values of orbital semi-major axis, a (in km)
‘2’ for explicit orbital semi-major axis, a input
*/

If InpCatAx = 1 Then
Call Module_3 VisTim_by_ax_byRange()
Else
Call Module_3 VisTim_by_ax_byExpInp()
EndIf
Return

End Module_3 VisTim_by_ax ()

Module_3 VisTim_by_ax_byRange ()

Input axmn // input lowest value of a in km

Input axmx // input highest value of a in km

Input Dmx // input number of subdivisions of the range axmn to axmx

Declare the array **ax**[Dmx] // array **ax**[Dmx] stores the 1,2,3,...Dmx orbital semi-major axis values

Declare the array VtHeS[Dmx] // array VtHeS [Dmx] holds the 1,2,3,... Dmx visibility times in seconds

Declare the array VtHeM[Dmx] // array VtHeM [Dmx] holds the 1,2,3,... Dmx visibility times in minutes

Declare the array VtHeH[Dmx] // array VtHeH [Dmx] holds the 1,2,3,... Dmx visibility times in HOUR

Input ε // input minimum elevation angle restriction value in radian

Input e //input eccentricity
For M = 1 , M ≤ Dmx, M = M + 1
$ax[\mathrm{M}] = M\left(\frac{\mathrm{elmx-elmn}}{\mathrm{Dmx}}\right)$
VtHeS[M] = compute_VT (e, $ax[\mathrm{M}]$, ε]) // Call the Module that computes the Mth visibility time when e , a and ε are given and the result is in seconds)
VtHeM[M] = $\left(\frac{\mathrm{VtHeS[M]}}{60}\right)$
VtHeH[M] = $\frac{\mathrm{VtHeS[M]}}{3600}$
Output M, e, **ax**[M], ε , VtHeS[M], VtHeM[M], VtHeH[M]
Next For Loop
End For Loop
End Module_3 VisTim_by_ax_byRange ()

Module_3 VisTim_by_ax_byExpInp()
Input Dmx // Dmx is the number of different orbital semi-major axis, a (in km) that the user want to consider
Declare the array **ax**[Dmx] // array **ax**[Dmx] stores the 1,2,3,...Dmx orbital semi-major axis values
Declare the array VtHeS[Dmx] // array VtHeS [Dmx] holds the 1,2,3,... Dmx visibility times in seconds
Declare the array VtHeM[Dmx] // array VtHeM [Dmx] holds the 1,2,3,... Dmx visibility times in minutes
Declare the array VtHeH[Dmx] // array VtHeH [Dmx] holds the 1,2,3,... Dmx visibility times in HOUR
Input ε // input minimum elevation angle restriction value in radian
Input e // input eccentricity
For M = 1 , M ≤ Dmx, M = M + 1
Input **ax**[M] // Input the Mth orbital semi-major axis, a (in km)
VtHeS[M] = compute_VT (e, **ax**[M], ε) // Call the Module that computes the Mth visibility time when e , a and ε are given and the result is in seconds)
VtHeM[M] = $\left(\frac{\mathrm{VtHeS[M]}}{60}\right)$
VtHeH[M] = $\frac{\mathrm{VtHeS[M]}}{3600}$
Output M, **e**, **ax**[M], ε , VtHeS[M], VtHeM[M], VtHeH[M]
Next For Loop
End For Loop
Return
End Module_3 VisTim_by_ax_byExpInp()

Module_4 VisTim_by_To ()
Input InpCatTo /* Choose the input category:
'1' for range of values of orbital period, To (in seconds)
'2' for explicit orbital period, To input
*/
If InpCatTo = 1 Then
Call Module_4 VisTim_by_To_byRange()
Else
Call Module_4 VisTim_by_To_byExpInp()
EndIf
Return
End Module_4 VisTim_by_To ()

Module_4 VisTim_by_To_byRange ()
Input Tomn // input lowest value of To in seconds
Input Tomx // input highest value of To in seconds

Input Dmx // input number of subdivisions of the range Tomn to Tomx
Declare the array To[Dmx] // array **To**[Dmx] stores the 1,2,3,...Dmx orbital period values
Declare the array **ax**[Dmx] // array **ax**[Dmx] stores the 1,2,3,...Dmx orbital semi-major axis values
Declare the array VtHeS[Dmx] // array VtHeS [Dmx] holds the 1,2,3,... Dmx visibility times in seconds
Declare the array VtHeM[Dmx] // array VtHeM [Dmx] holds the 1,2,3,... Dmx visibility times in minutes
Declare the array VtHeH[Dmx] // array VtHeH [Dmx] holds the 1,2,3,... Dmx visibility times in HOUR
Input ε // input minimum elevation angle restriction value in radian
Input e // input eccentricity
For M = 1 , M ≤ Dmx, M = M + 1

$$\text{To}[\text{M}] = M\left(\frac{\text{Tomx} - \text{Tomn}}{\text{Dmx}}\right)$$

$$\mathbf{ax}[\text{M}] = \sqrt[3]{\left(\mu\left(\frac{To[\text{M}]}{2\pi}\right)^2\right)}$$

VtHeS[M] = compute_VT (e, ax[M], ε]) // Call the Module that computes the Mth visibility time when **e** , **a** and ε are given and the result is in seconds)

$$\text{VtHeM[M]} = \left(\frac{\text{VtHeS[M]}}{60}\right)$$

$$\text{VtHeH[M]} = \frac{\text{VtHeS[M]}}{3600}$$

Output M, e, **ax**[M], To[M], ε , VtHeS[M], VtHeM[M], VtHeH[M]
Next For Loop
End For Loop
End Module_4 VisTim_by_To_byRange ()

Module_4 VisTim_by_To_byExpInp()

Input Dmx // Dmx is the number of different orbital period, To (in seconds) that the user want to consider
Declare the array To[Dmx] // array **To**[Dmx] stores the 1,2,3,...Dmx orbital period values
Declare the array **ax**[Dmx] // array **ax**[Dmx] stores the 1,2,3,...Dmx orbital semi-major axis values
Declare the array VtHeS[Dmx] // array VtHeS [Dmx] holds the 1,2,3,... Dmx visibility times in seconds
Declare the array VtHeM[Dmx] // array VtHeM [Dmx] holds the 1,2,3,... Dmx visibility times in minutes
Declare the array VtHeH[Dmx] // array VtHeH [Dmx] holds the 1,2,3,... Dmx visibility times in HOUR
Input ε // input minimum elevation angle restriction value in radian
Input e // input eccentricity
For M = 1 , M ≤ Dmx, M = M + 1
Input **To**[M] // Input the Mth orbital period, To (in seconds)

$$\mathbf{ax}[\text{M}] = \sqrt[3]{\left(\mu\left(\frac{To[\text{M}]}{2\pi}\right)^2\right)}$$

VtHeS[M] = compute_VT (e, **ax**[M], ε) // Call the Module that computes the Mth visibility time when **e** , **a** and ε are given and the result is in seconds)

$$\text{VtHeM[M]} = \left(\frac{\text{VtHeS[M]}}{60}\right)$$

$$\text{VtHeH[M]} = \frac{\text{VtHeS[M]}}{3600}$$

Output M, **e**, **ax**[M],**To**[M], ε , VtHeS[M], VtHeM[M], VtHeH[M]
Next For Loop
End For Loop
Return
End Module_4 VisTim_by_To_byExpInp()

3. The details of the selected case study highly eccentric orbit satellites

The details of the selected five case study highly eccentric orbit satellites are given in Table 1 while the visual presentation of their orbital shapes are presented in Figure 1, Figure 2, Figure 3, Figure 4 and Figure 5. The orbital views in Figure 1 to Figure 5 are obtained online from https://www.heavens-abovea.com/. The shape of the orbits of the listed satellites are elliptical which is due t the high values of their eccentricity.

Table 1 The details of the selected case study highly eccentric orbit satellites

Satellite Name , Norad ID and application category	Semi major axis in km	Eccentricity	Source
I. SPIRALE B • 33752 • Military	21136	0.6867818	https://www.n2yo.com/satellite/?s=33752
II. IMAGE • 26113 • Space & Earth Science	29813	0.7348416	https://www.n2yo.com/satellite/?s=26113
III. XMM • 25989 • Space & Earth Science	66932	0.5960958	https://www.n2yo.com/satellite/?s= 25989
IV. INTEGRAL • 27540 • Space & Earth Science	81110	0.8892668	https://www.n2yo.com/satellite/?s=27540
V. MMS 4 • 40485 • Space Science	97697	0.8733602	https://www.n2yo.com/satellite/?s=40485

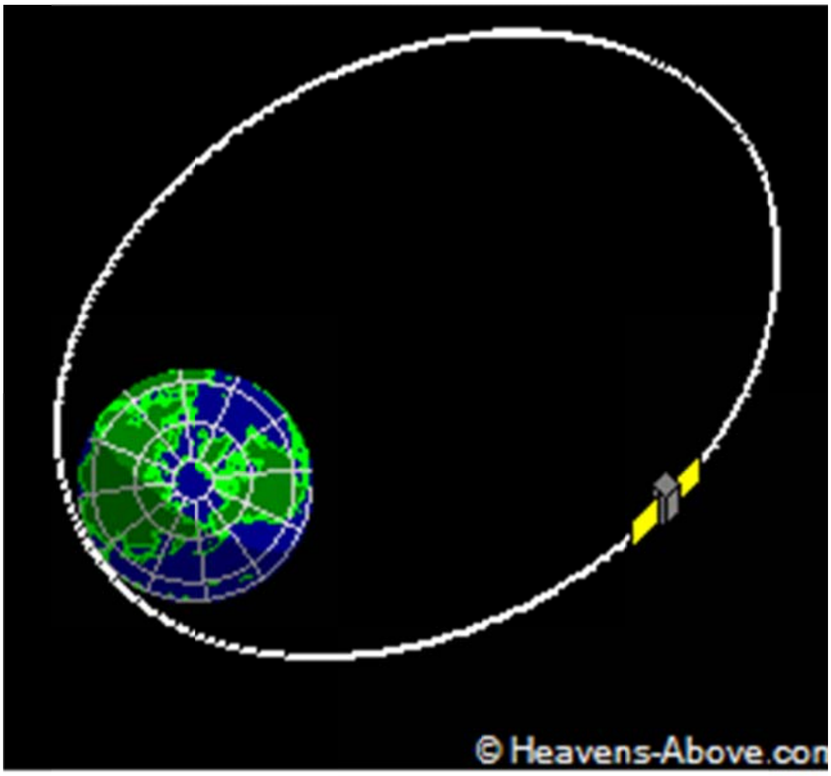

Figure 1 The orbital view of SPIRALE B satellite which has eccentricity value of 0.6867818 [Source: https://www.heavens-above.com/]

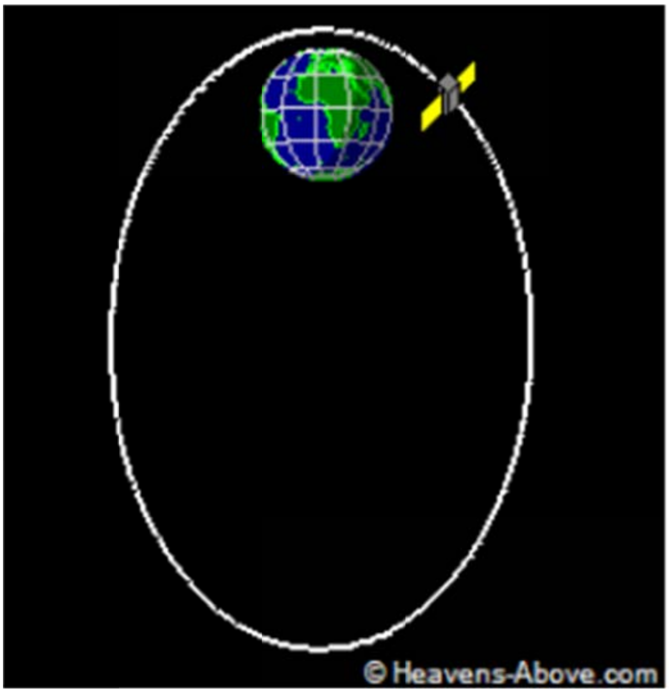

Figure 2 The orbital view of IMAGE satellite which has eccentricity value of 0.7348416 [Source: https://www.heavens-above.com/]

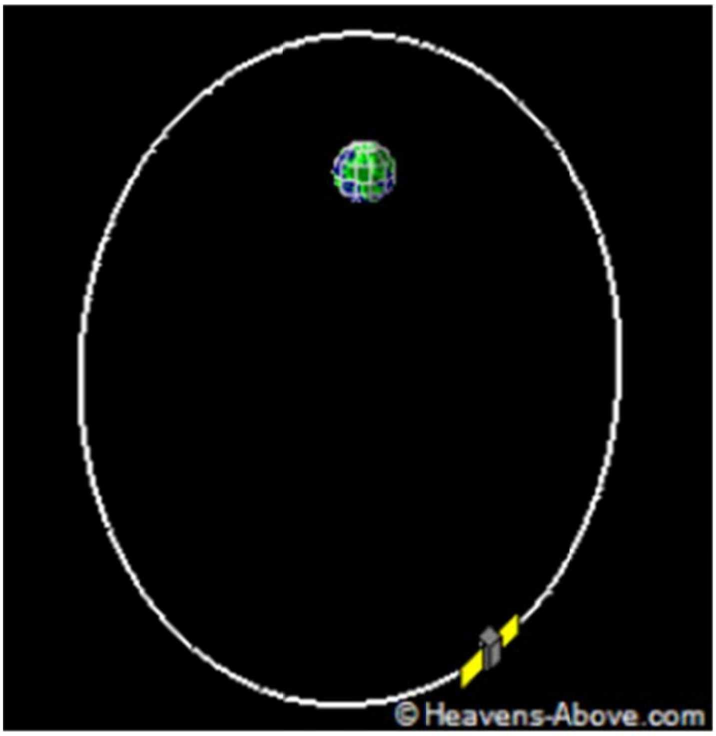

Figure 3 The orbital view of XMM satellite which has eccentricity value of 0.5960958 [Source: https://www.heavens-above.com/]

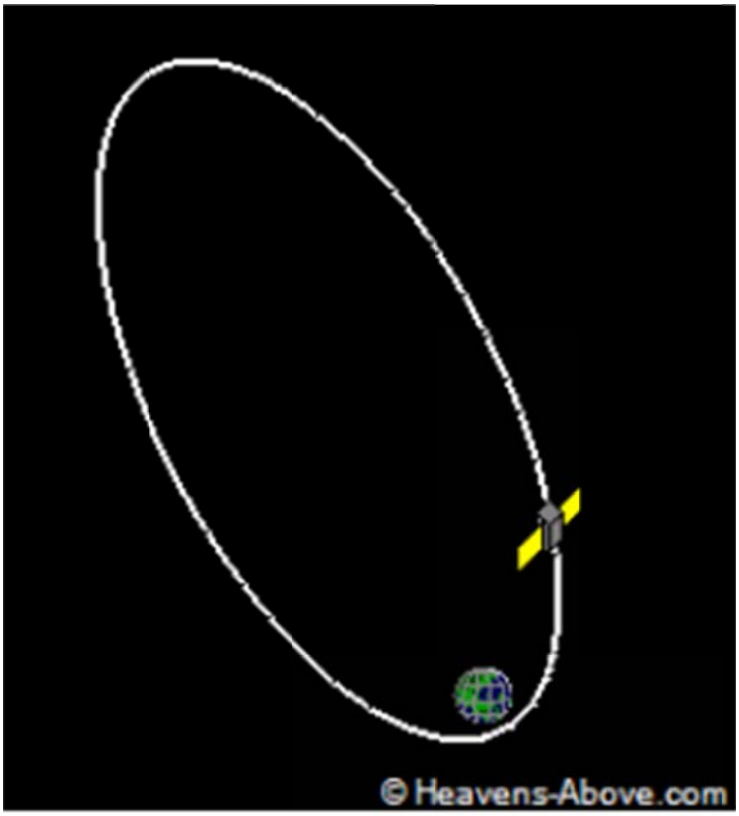

Figure 4 The orbital view of INTEGRAL satellite which has eccentricity value of 0.8892668 [Source: https://www.heavens-above.com/]

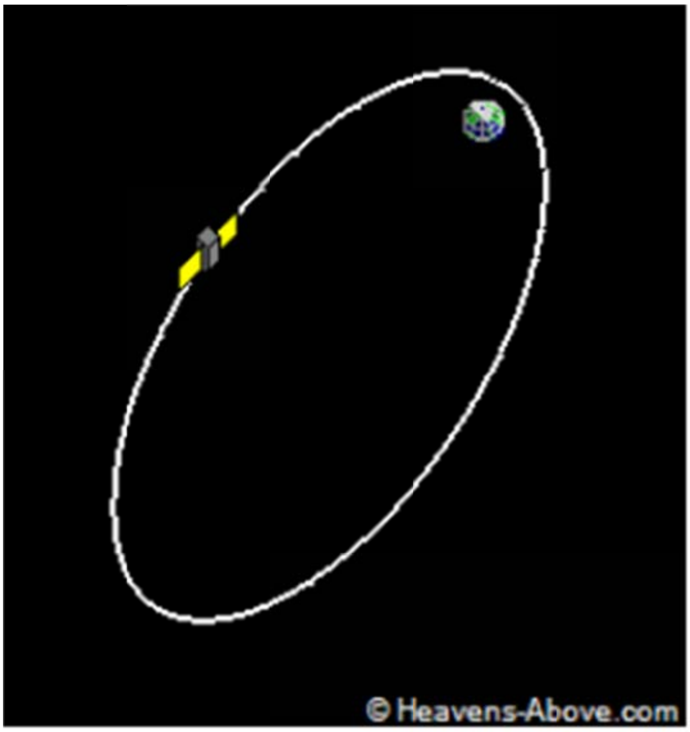

Figure 5 The orbital view of MMS 4 satellite which has eccentricity value of 0.8733602 [Source: https://www.heavens-above.com/]

4.0 Results and discussion

The results for the analysis of the effect of eccentricity (e) on the visibility time are given in Table 2 for semi-major axis, a = 21,136 km and minimum elevation angle, $\varepsilon min = 0°$. Also, the results for the analysis of the effect of minimum elevation angle, εmin time are given in Table 3, for semi-major axis, a = 21,136 km and eccentricity, e = 0.735. Again, the results for the analysis of the effect of semi-major Axis, a are given in Table 4 for minimum elevation angle, εmin = 0° and eccentricity, e = 0.735.

Table 2 The results for the analysis of the effect of eccentricity (e) on the visibility time for semi-major axis, a = 21,136 km and minimum elevation angle, εmin =0°

S/N		Minimum elevation angle, εmin (°)	Eccentricity, e	Orbital Semi-Major Axis, a (km)	Orbital Period, To (hour)	Visibility time in hour	Percentage of visibility time per orbital period (%)
1	XMM	0	0.596	21,136.000	8.495	7.268	85.563
2		0	0.617	21,136.000	8.495	7.358	86.623
3		0	0.638	21,136.000	8.495	7.446	87.661
4		0	0.659	21,136.000	8.495	7.533	88.676
5	SPIRALE B	0	0.687	21,136.000	8.495	7.644	89.988
6		0	0.708	21,136.000	8.495	7.725	90.943
7		0	0.729	21,136.000	8.495	7.804	91.871
8	IMAGE	0	0.735	21,136.000	8.495	7.827	92.139
9		0	0.756	21,136.000	8.495	7.902	93.028
10		0	0.777	21,136.000	8.495	7.975	93.884
11		0	0.798	21,136.000	8.495	8.045	94.706
12		0	0.819	21,136.000	8.495	8.112	95.491
13	MMS 4	0	0.873	21,136.000	8.495	8.269	97.347
14		0	0.878	21,136.000	8.495	8.282	97.501
15	INTEGRAL	0	0.889	21,136.000	8.495	8.310	97.825

Table 3 The results for the analysis of the effect of minimum elevation angle, εmin for semi-major axis, a = 21,136 km and eccentricity, e = 0.735

S/N	Minimum elevation angle, εmin (°)	Eccentricity, e	Orbital Semi-Major Axis, a (km)	Orbital Period, To (hour)	Visibility time in hour	Percentage of visibility time per orbital period (%)
1	0	0.735	21,136	8.495	7.827	92.139
2	1	0.735	21,136	8.495	7.740	91.115
3	2	0.735	21,136	8.495	7.653	90.091
4	3	0.735	21,136	8.495	7.566	89.068
5	4	0.735	21,136	8.495	7.479	88.044
6	5	0.735	21,136	8.495	7.392	87.020
7	6	0.735	21,136	8.495	7.305	85.996
8	7	0.735	21,136	8.495	7.218	84.972
9	8	0.735	21,136	8.495	7.131	83.949
10	9	0.735	21,136	8.495	7.044	82.925
11	10	0.735	21,136	8.495	6.957	81.901
12	11	0.735	21,136	8.495	6.870	80.877
13	12	0.735	21,136	8.495	6.783	79.854
14	13	0.735	21,136	8.495	6.696	78.830
15	14	0.735	21,136	8.495	6.609	77.806

Table 4 The results for the analysis of the effect of semi-major Axis, a for minimum elevation angle, εmin = 0° and eccentricity, e = 0.735

S/N	Satellite with applicable Semi-Major Axis, a (km)	Minimum elevation angle, εmin (°)	Eccentricity, e	Orbital Semi-Major Axis, a (km)	Orbital Period, To (hour)	Visibility time in hour	Percentage of visibility time per orbital period (%)
1	SPIRALE B	0	0.735	21,136	8.495	7.827	92.139
2		0	0.735	26,605	11.996	11.053	92.139
3	IMAGE	0	0.735	29,813	14.230	13.112	92.139
4		0	0.735	35,282	18.321	16.880	92.139
5		0	0.735	40,750	22.741	20.953	92.139
6		0	0.735	46,219	27.469	25.309	92.139
7		0	0.735	51,688	32.486	29.932	92.139
8		0	0.735	57,156	37.775	34.805	92.139
9	XMM	0	0.735	66,932	47.870	44.106	92.139
10		0	0.735	72,401	53.855	49.621	92.139
11		0	0.735	77,869	60.070	55.348	92.139
12	INTEGRAL	0	0.735	81,110	63.859	58.839	92.139
13		0	0.735	86,579	70.425	64.889	92.139
14		0	0.735	92,047	77.201	71.132	92.139
15	MMS 4	0	0.735	97,697	84.417	77.781	92.139

The graph of orbital Period, To (hour) and Visibility time in hour versus eccentricity for semi-major axis, a = 21136 km is given in Figure 6 while the graph of percentage of visibility time per orbital period (%) versus eccentricity for semi-major axis, a = 21136 km is given in Figure 7. The results in Figure 6 and Figure 7 show that one, both visibility time and percentage of visibility time per orbital period increase with increase in eccentricity, and two, both visibility time and percentage of visibility time per orbital period are higher at minimum elevation angle, εmin = 0º than their corresponding values at εmin =5º. In essence, both visibility time and percentage of visibility time per orbital period decreases with increase in minimum elevation angle restriction. In any case, the orbital period is not affected by the eccentricity.

The graph of Visibility time in hour versus minimum elevation angle, εmin (°) for semi-major axis, a = 21136 km is given in Figure 8 while the graph of percentage of visibility time per orbital period (%) versus minimum elevation angle, εmin (°) for semi-major axis, a = 21136 km is given in Figure 9. The results in Figure 8 and Figure 9 show that one, both visibility time and percentage of visibility time per orbital period decrease with increase in minimum elevation angle restriction, and two, both visibility time and percentage of visibility time per orbital period are higher at eccentricity, e = 0.735 than their corresponding values at eccentricity, e = 0.889. In essence, both orbital Period and percentage of visibility time per orbital period increase with increase in eccentricity. In any case, the orbital period is not affected by the minimum elevation angle, εmin.

The graph of Visibility time in hour versus orbital semi-major axis is given in Figure 10 while the graph of percentage of visibility time per orbital period (%) versus orbital semi-major axis is given in Figure 11. The results in Figure 10 and Figure 11 show that one, orbital period , visibility time and percentage of visibility time per orbital period increase with increase in orbital semi-major axis and two, both visibility time and percentage of visibility time per orbital period are higher at eccentricity, e = 0.735 than their corresponding values at eccentricity, e = 0.889. Again, this confirms the earlier result that both orbital Period and percentage of visibility time per orbital period increase with increase in eccentricity.

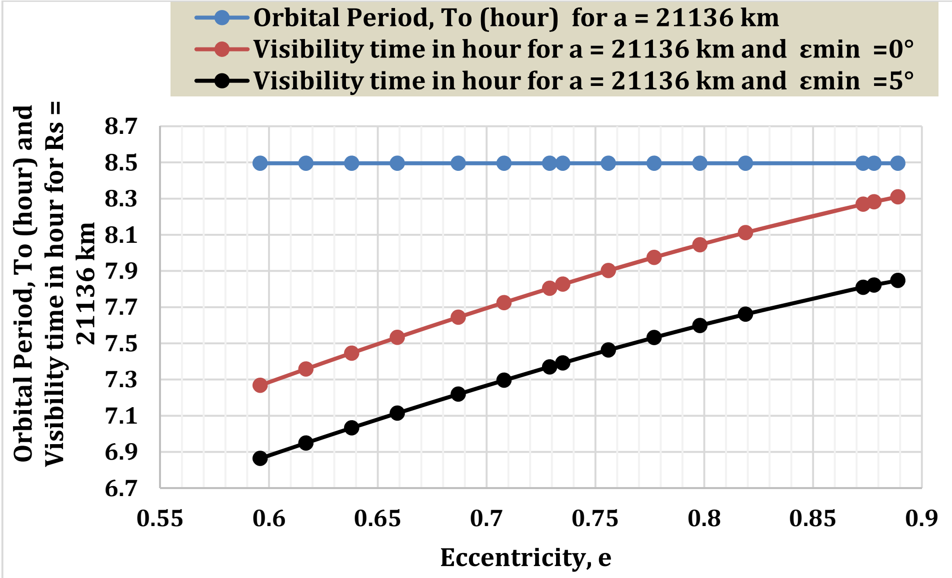

Figure 6 The graph of orbital Period, To (hour) and Visibility time in hour versus eccentricity for semi-major axis, a = 21136 km

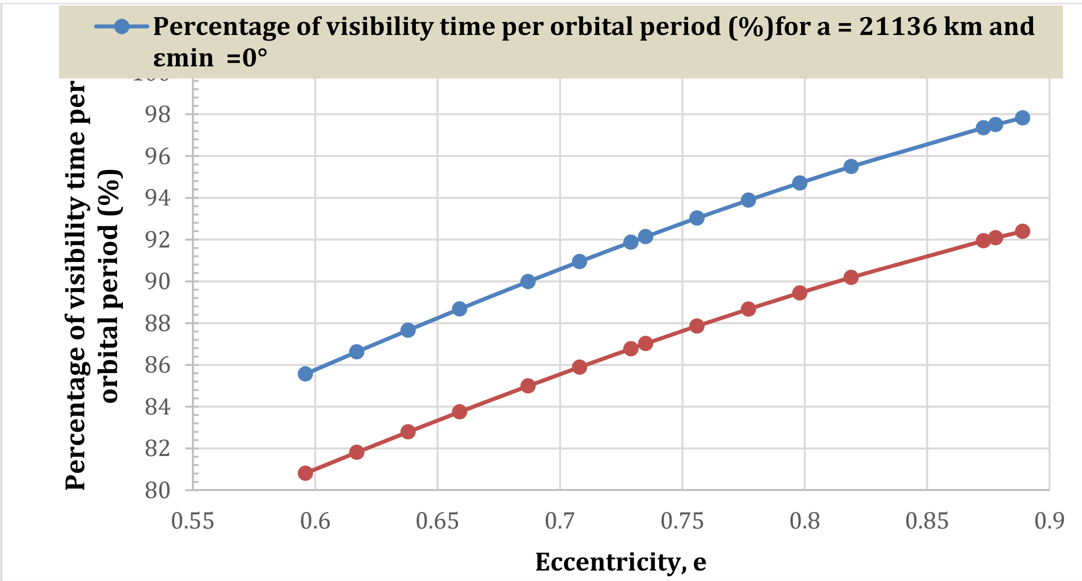

Figure 7 The graph of percentage of visibility time per orbital period (%) versus eccentricity for semi-major axis, a = 21136 km

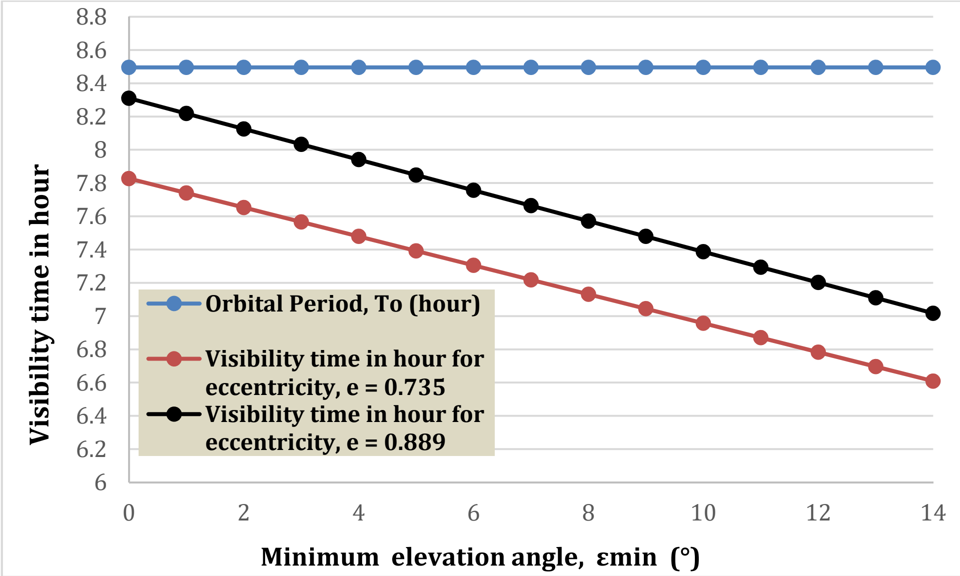

Figure 8 The graph of Visibility time in hour versus minimum elevation angle, εmin (°) for semi-major axis, a = 21136 km

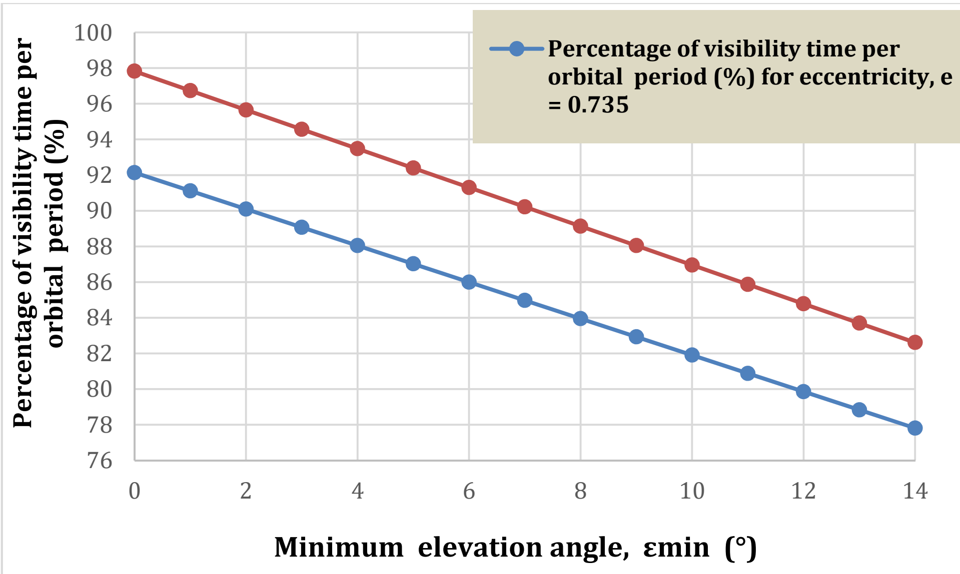

Figure 9 The graph of percentage of visibility time per orbital period (%) versus minimum elevation angle, εmin (°) for semi-major axis, a = 21136 km

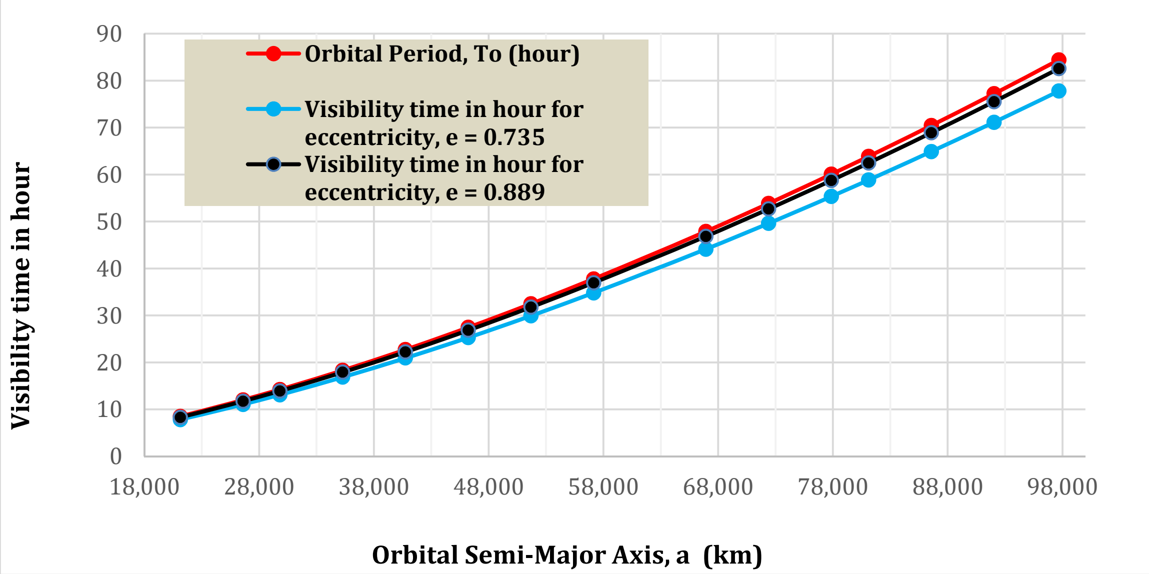

Figure 10 The graph of Visibility time in hour versus orbital semi-major axis, a

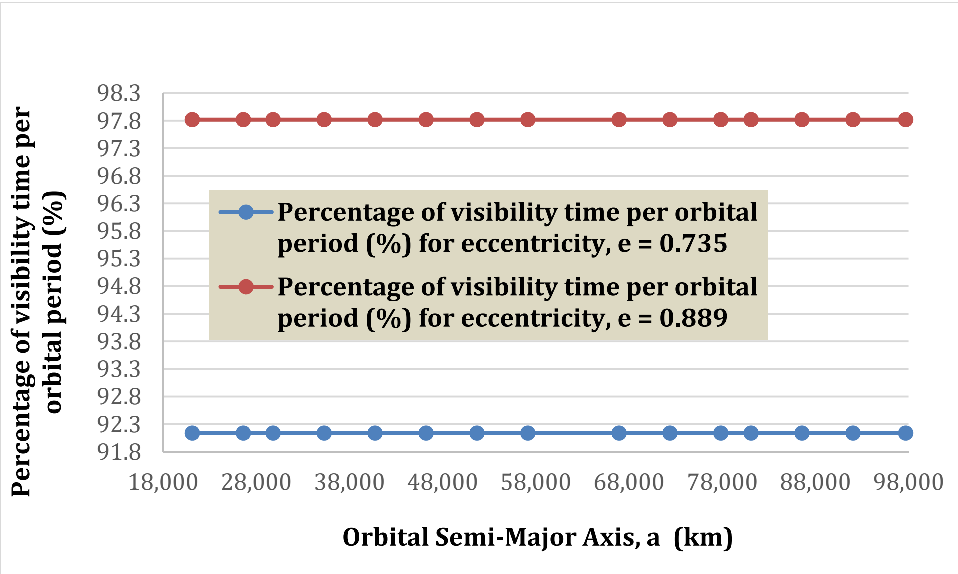

Figure 11 The graph of percentage of visibility time per orbital period (%) versus orbital semi-major axis, a

4. Conclusion

A modular program design is employed in the development of Visual Basic for Application-based program for conducting parametric analysis of the visibility time of satellites with high eccentricity values. Such orbits are elliptical and their visibility times are significantly affected by the value of the eccentricity. The paper presented the analytical expression and the program design using pseudocode approach. Then, five highly elliptical orbit satellites were used for numerical examples. The results are presented in tables and graph plots.

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