A General Decentralized Repeat Inspection Plan For Dependent Multi-Characteristic Critical Components

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***Abstract*—Inspections plans play an important role in quality control. They insure certain standard of quality. A multi-characteristic critical component is defined as a compenent when it fails it causes disaster or a very high cost. Such components could be part of a gas ignition system, an aircraft, a space shuttle or a special weapon system. In many situations the failure of the characteristics are statistically dependent. In this paper a mathematical model is developed for multi-characteristic components where the failure of characteristics are statistically dependent using the decentralized plan proposed by Duffuaa and Al-Najjar [3]. In this model inspectors commit type I error (classifying a non-defective characteristic as defective) and type II errors (classifying a defective characteristic as non-defective). The model minimized total expected cost per accepted compoent. The total cost consists of the cost of type I, type II errors and the cost of inspection. An algorithm is proposed to determine the optimal number of repeat inspections for each characteristic to minimize the total expected cost per accepted component. An example is presented to demonstrate te model. The example is run for both indpendent and dependent failures rates and a difference in the plan performance measures is observed. Three measures are used in the example include average total insection (ATI) and average outgoing quality (AOQ) is observed.**

Keywords—Multi-characteristic, Repeat inspection, Expected cost. Dependent failure.

1.0 Introduction

A Multi-characteristic critical component is a critical component with several characteristics. A component is critical if upon failure it causes a disaster or very high cost. Examples in the literature have provided components with up to fourteen characteristics. Such components could be a part of a gas ignition system, a space shuttle, or a special weapon system. To ensure failure free components repeat inspection is instituted. The repeat inspection is performed on such components because inspection is not error free. Inspectors usually commit two types of errors. Type I error (classifying a non-defective characteristic as defective) and type II error (classifying a defective characteristic as non-defective). In critical multi-characteristic components type II is more series and as such repeat inspection is performed. The literature has several models for determining optimal number of repeat inspection that minimize total expected cost [1, 2, 3].

The total expected cost per accepted component consists of inspection cost, cost of type I error and cost of type II error. Most of the models in the literature assume that the characteristic defective rates are statistically independent except the model given in [2]. Duffuaa and Al-Najjar proposed a new decentralized inspection plan [3]. The new decentralized plan is used as the basis for developing the model in this paper.

Raouf et al. [1] developed a model for determining the optimal number of repeat inspections for multicharacteristic components to minimize the total expected cost per accepted component due to Type I error, Type II error and cost of inspection. Garcia-Diaz et al. [4] presented a dynamic programming (DP) model for repeat 100% inspection. Elmaghraby [5] further analysed the model of Garcia-Diaz et al. and presented an alternative condition for the applicability of the *DP* model. Jaraeidi et al [6] presented a model to determine the average outgoing quality (*AOQ)* for a product which has multiple quality characteristics and which is subjected to multiple 100% inspections where the inspection is subjected to errors. Lee [7] presented a simplified version of the cost-minimization model developed by Raouf et al. [1] to capture the cost implication of the false rejection, false acceptance and inspection of the components. Optimality of the sequence of the characteristics to be inspected was also obtained. Duffuaa and Raouf [8] developed three mathematical optimization models for multicharacteristic repeat inspection. The first model (cost minimization model) minimizes the total cost due to inpsections, Type I error and Type II error to determine the optimal number of repeat inspections. The second model (probability minimization model) minimizes the probability of accepting a defective component. The third model (satisfying model) determines a satisfying solution by specifying an upper limit for total inspection cost and for the probability of accepting a defective component. Duffuaa and Raouf [9] established an optimal rule for sequencing characteristics for inspection in the plan proposed by Raouf et al [1]. Duffuaa and Nadeem [2] developed an extension of the model proposed in Raouf et al [1] for components whose characteristics's defective rates are statistically dependent. Duffuaa and Al-Najjar [3] proposed a new inspection plan for critical multicharacteristic components. They proposed an algorithm to determine the optimal number of repeat inspections and sequence characteristics for inspection in order to minimizes the total expected cost. The literaure review has shown that this plan has not been utlized in modeling multi-characteristic critical compoents where the characteristic's defective rate are statistically dependent.

The rest of the paper is organized as follow: Section 2 states the problem, followed by the proposed model in Section 3. An algorithm to solve the model together with an illustrative example is provided in section 4 and Section 5 concludes the paper.

2. Statement of the Problem

The problem under consideration is to ensure almost defect free ommoents because the type of comments we are dealing with in this paper are critical and their failures after ounted on te system where they are mounted causes disastor or extremly high costs. On the other hand inspectors commit type I and type II errors. His means they can reject a good compoent (type I error) or accept a defective compoents (type II error). The harm from rejecting a good compoent is far much less than accepting a defective compoent.In order to minimize both type of errors repeat inspection is instituted. The question how many inspections to conduct before accepting the compoent. This will be determined based on cost minimization. In such inspection plans there several costs that are icurred. These are the cost of inspection, cost of type I error and cost of type 11 errors. The characteristics failure rates are assumed to be statistically dependent.

It is also assumed that the rate of type I and II errors, cost of inspection, charactristics failure rates are known or we have very accurate estmates for them.

3.0 Model Development

The model is developed for components with several characteristics which are statistically dependent. A component is accepted if all of its characteristics meet the quality specifications. We denote the random variable X_i which takes the value 0 if characteristics i is defective and 1 if it is non-defective. The joint probability density function of the multivariate random variable $X = (x_1, x_2,., x_N)$ is assumed to be known. The value of E_{1i} and E_{2i} are also assumed to be known or estimated from existing data. The expected total cost involves the cose of false acceptance, the cost of false rejection and the cost of inspection. The inspection plan is concepulaized as shown in figure 1 where each characteristic is inspected equal number of times. In the nomenclature given below, i ranges from 1 to N and j ranges from 1 to n.

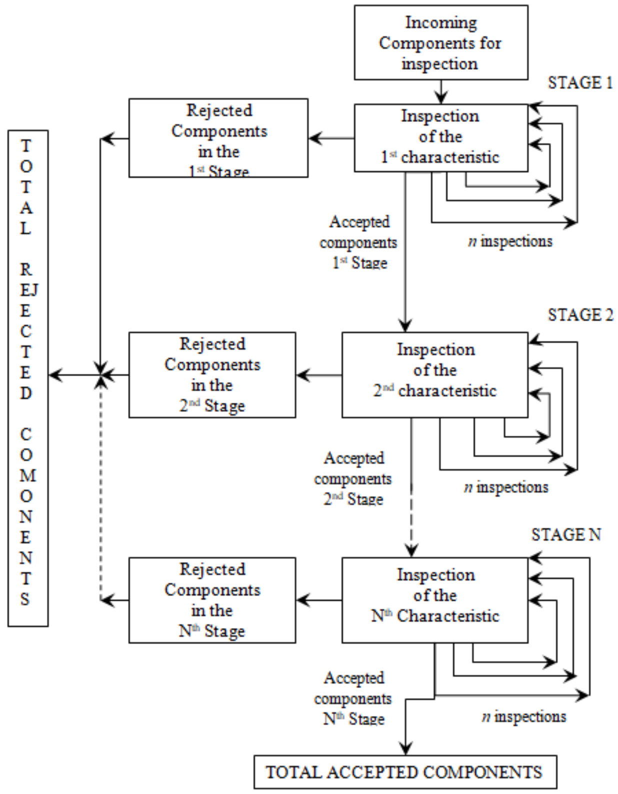

Figure 2.1: The Inspection Plan

3.1 Nomenclature

M	Number of components to be inspected.
M_i	Number of components entering i-th stage of inspection.
$M_{i,j}$	Number of components entering the j-th cycle of stage i.
N	Number of charachteirstics in each component to be inspected.
n	Optimal number of repeat inspections.
C_i	Cost of inspection of characteristic i.
C_a	Cost of acceptance per bad component.
C_r	Cost of rejection per good component.
P_i	Probability of the i-th characteristic being defective on entering the isnepction.
$P_{i,j}$	Probability of the i-th characteristic in the sequence of inspection being defective on entering the j-th cycle of inspection.
E_{1i}	Probability of classifying the i-th non-defective characteristic in the sequence of inspection as defective (Type I error).
E_{2i}	Probability of classifying the i-th defective characteristic in the sequence of inspection as non-defective (Type II error).
PG	Probability of a component being non-defective on entering the inspection process.
$PG_{i,j}$	Probability of the component being non-defective on entering the j-th cycle of the i-th stage of inspection.
$FR_{i,j}$	Expected number of falsely rejected

	components in the *j*-th cycle of the *i*-th stage.
$FA_{i,j}$	Expected number of falsely accepted components in the *j*-th cycle of the *i*-th stage.
$CA_{i,j}$	Expected number of correctly accepted components in the *j*-th cycle of the *i*-th stage.
$R_{i,j}$	Rate of rejection of components due to the *i*-th characteristic in the sequence of inspection of the *j*-th cycle.
A_i	Expected number of accepted components in the *i*-th stage.
CFR_i	Cose of false rejection in the *i*-th stage.
CFA_i	Cose of false acceptance in the *i*-th stage.
CI_i	Cost of inspection in the *i*-th stage.
TCFR	Total cost of false rejection.
TCFA	Total cost of false acceptance.
TCI	Total cose of inspection.
TA	Total number of accepted components.
$E(tc)\|_i$	Expected total cost per accepted component after the *i*-th stage of inspection.
X_i	A discrete random variable which takes value 0 if characteristic *i* is defective and 1 if it is non-defective.
$P_i(x_i)$	The marginal probability mas function of the random variable X_i.
$P_i(0)$	Probability of the *i*-th characteristic being defective on entering the inspection process.
$^jP_i(0)$	Probability of the *i*-th characteristic being defective on entering the *j*-th inspection cycle.
$P(x_1, x_2, ., x_N)$	The joint probability mass function of the random variables X_i, *i=1,2,,., N.*
$iP(x_1, x_2, ., x_N)$	The joint probability mass function of the random variables X_i for the component entering the *i*-th stage of inspection.

3.2Basic Relationships of the Model

At the start of the inspection the joint probability mass function (*j.p.m.f.*) of the random variables (X_1, X_2, ., X_N) is assumed to be know. So, using the *j.p.m.f.* we can obtain the individual marginal probability mass function for each characteristic.

$$P_i(x_i) = \sum_{x_1}\sum_{x_2}\sum_{x_3}...\sum_{x_{i-1}}\sum_{x_{i+1}}...\sum_{x_N} P(x_1, x_2, \ldots, x_n)$$

(1)

The marginal mass functions will vary from cycle to cycle

$^1P_i(0) = P_i(0)$ (2)

Using Bayes theorem

$$^2P_i(0) = \frac{P_i(0)E_{2i}}{[P_i(0)E_{2i} + (1-P_i(0))(1-E_{1i})]} \quad (3)$$

$^2P_i(1) = 1 - {}^2P_i(0)$ (4)

The updated values for the individual random variable marginal mass function can be obtained using equation 3 and 4. In general, the marginal probability mass function for the *i*-th characteristic at the *j*-th cycle of inspection is:

$$^jP_i(0) = \frac{^{j-1}P_i(0)E_{2i}}{[^{j-1}P_i(0)E_{2i} + (1-{}^{j-1}P_i(0))(1-E_{1i})]} \quad (5)$$

$^jP_i(1) = 1 - {}^jP_i(0)$ (6)

Owing to the inspection the joint and the marginal mass function must be updated after the *n* inspection of the characteristics at each stage. Because of the statistical dependency between characteristic *i* and other characteristics, the marginal of the other characteristics must be updated prior to inspecting them. The updated values of the joint probability mass function will be obtained using Bayes theorem. After inspecting characteristic *i*, *n* times at the first stage of inspection the rule for updating the joint probability mass function is

$^1P(x_1, x_2, ., x_N) = P(x_1, x_2, ., x_N)$ (7)

$$^2P(x_1, x_2, \ldots, x_n) = {}^1P(x_1, x_2, \ldots, x_n)\frac{^{n+1}P_i(x_i)}{^1P_i(x_i)}$$

i.e. we multiply the old joint probability mass function by the updated marginal mass funtion for characteristic *i* (the characteristic which is just inspected *n* times at the first stage) and divided by the old marginal mass function of the inspected characteristic. It can be seen from Bayes theorem that the updated function is a probability mass function. After obtaining the updated joint probability mass function we can find the marginal for each characteristic. Then we can inspect the second characteristic *n* times and so on until we inspect all characteristics *n* times. At the end of each stage, we can compute the probability of the component being non-defective which is given by:

$$PG_{1,n+1} = {}^1P(1,1,1,\ldots,1)\frac{^{n+1}P_i(1)}{^1P_i(1)} \quad (8)$$

The probability of a component being non-defective entering the *n*-th cycle of inspection of the *N*-th characteristic is given by

$$PG_{N,n} = {}^NP(1,1,1,\ldots,1)\frac{^nP_i(1)}{^1P_i(1)} \quad (9)$$

If no inspection is performed we incur only the cost of false acceptance

$$E(tc)\big|_{j=0} = C_a\left[1 - P(1,1,1,\ldots 1)\right] \quad (10)$$

The expected total cost per accepted component, after inspecting each characteristic *n* times will be

$$E(tc)\Big|_{j=0} = \left[TCFR + TCFA + TCI\right]/TA \quad (11)$$

Results needed to compute expected total cost for each cycle are given below for the Nth cycle:

The total number of accepted components after completing N stages of inspection, i.e. after inspecting the N-th characteristic is given as:

$$A_N = M\left[\prod_{k=1}^{N-1}\prod_{j=1}^{n}\left[P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})\right]\right]\times\left[\prod_{j=1}^{n-1}\left[P_{n,j}E_{2n} + (1-P_{n,j})(1-E_{1n})\right]\right]$$
$$\times\left[P_{N,n}E_{2N} + (1-PG_{N,n} - P_{N,n})(1-E_{1N})\right] + M\left[PG\prod_{k=1}^{N}(1-E_{1k})\right]$$

(12)

The cost of false rejection at each stage *i, i* = 1, .,*N*, is given as:

$$CFR_i = \left[C_r \times M \times PG \times E_{1i}\right]\left[\prod_{k=1}^{i-1}(1-E_{1k})^n\right]\times\left[\sum_{k=1}^{n}(1-E_{1i})^{k-1}\right]$$

(13)

The cost of false acceptance after completing *N* stages of inspection is given as:

$$CFA_N = C_a M\left[\prod_{k=1}^{N-1}(\prod_{j=1}^{n}\left[P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})\right]\right]$$
$$\times\left[\prod_{j=1}^{n-1}\left[P_{N,j}E_{2N} + (1-P_{N,j})(1-E_{1N})\right]\right]$$
$$\times\left[P_{N,n}E_{2N} + (1-PG_{N,n} - P_{N,n})(1-E_{1N})\right]$$

(14)

The cost of inspection at each stage *i, i,* = 1, ., N, is given as:

$$CI_i = C_i M \times\left[\prod_{k=1}^{i-1}\prod_{j=1}^{n}\left\{P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})\right\}\right]$$
$$\times\left[\sum_{k=1}^{n}\left\{\prod_{j=1}^{k-1}\left[P_{i,j}E_{2i} + (1-P_{i,j})(1-E_{1i})\right]\right\}\right]$$

(15)

Now, in order to determine the general expression for the expected total cost per accepted component, we must determine the total cost of false rejection *TCFR*, the total cost of false acceptance *TCFA*, the total cost of inspectin *TCI* and the total number of components finally acceted *TA*.

$$TCFR = \sum_{i=1}^{n} CFR_i \quad (16)$$

$$TCFA = CFA_N = CaFA_{N,n} \quad (17)$$

$$TCI = \sum_{i=1}^{N} CI_i \quad (18)$$

$$TA = A_N = FA_{N,n} + CA_{N,n} \quad (19)$$

$$E(tc)\Big|_{j=n} = \frac{TCFR + TCFA + TCI}{TA} \quad (20)$$

The objective is to find the value of *n* which provides the minimum of $E(\text{tc})|_{j=n}$.The probability of a component being non-defective entering the *n*-th cycle of inspection of the *N*-the characteristic is given by

$$PG_{N,n} = \left[\prod_{i=1}^{N-1}(1-P_{i,n+1})\right](1-P_{N,n}) \quad (21)$$

Results needed to compute expected total cost

The total number of accepted components after completing *N* stages of inspection, i.e. after inspecting the *N*-the characteristic is given as:

$$M_{i,j} = M\left[\prod_{k=1}^{i-1}\prod_{j=1}^{n}\left[P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})\right]\right]$$
$$\times\left[\prod_{k=1}^{j-1}\left\{P_{i,k}E_{2i} + (1-P_{i,k})(1-E_{1i})\right\}\right]$$

(22)

Determing the Optimal Sequence of Inspection

The cost of inspection is influenced by the sequence in which the characteristics are ordered for inspection, i.e. the order of stages. The following rule provides the optimal sequence of inspection for the characteristics, when each characteristic is inspected *j* times.

$$Let\ r_i = \frac{C_i f_1(R_{i,j})}{1 - f_2(R_{i,j})} \quad i=1,2,.,N.\ j=1,2,.,n.\ (23)$$

where

$$R_{i,j} = P_{i,j(1-}E_{2i}) + 1(1-P_{i,j})E_{1,i}$$

$$f_1(R_{i,j}) = \sum_{j=1}^{n}\left[\prod_{k=1}^{j}(1-R_{i,k-1})\right]$$

$$f2(R_{i,j}) = \prod_{k=1}^{n}(1-R_{i,k})$$

At the end of each inspection cycle, the total cost due false rejection (TCFR), total cost due to false acceptance (TCFA) and cost of inspection (TC) are computed. We add all the costs compoents and divide by total accepted compoents (TA) as presented in equation (11).

4.0 Algorithm to determine the optimal number of repeat inspections

The algorithm consists of the ollowing steps:

STEP 1:Set *j* = 0, find $E(tc)\big|_{j=0}$ using equation (10)

STEP 2:Find the ratio of the characteristics according to equation (23). Inspect the characteristic with the lowest ratio *j* times.

STEP 3:Update the *j.p.m.f* of the characteristics remaining to be inspected using equation (8). Find the ratio of the characteristics remaining to be inspected using equation (23). Inspect the characteristic with the lowest ratio *j* times. Repeat this until all the characteristics are inspected.

STEP 4:Compute $^{j}P_i(0$, $^{j}P_i(1)$, $PG_{N,n}$, A_N, CFR_i, CFA_N, CI_i *for i* = 1,2, .,*N* from equations (5), (6), (9), (12), (13), (14), and (15), respectively.

STEP 5:Compute *TCFR, TCFA, TCI, TA* and $E(tc)_j$ from equations (16), (17), (18), (19) and (20), respectively.

STEP 6:If $E(tc)_j \langle E(tc)\big|_{j=1}$ set *j* = *j* +1 and go to SETP 2, otherwise STOP, *n* = *j* – 1.

Example:

A program is developed implementing the algorithms stated above. In the example, we have 100 components, each with three characteristics. The joint probability mass function for the defective rates is *P*(0, 0, 0) = 0.05, *P*(0, 0, 1) = 0.05, *P*(0, 1, 0) = 0.05, *P*(1, 1, 0) = 0.15, *P*(1, 0, 0) = 0.05, *P*(1, 0, 1) = 0.05, *P*(0, 1, 1) = 0.1, *P*(1, 1, 1) = 0.5. Other input values are, C_a = 100,000, C_r = 500, C_i = 100, E_{1i} = 0.01 and E_{2i} = 0.015. The results of the above example problem are given in Table 1 for the independent and the dependent cases. Three measures wereused to compare the results of the indpendent and the dependent case. The results in table 1 show some differences between the independent and the dependent cases in PG, ATI and AOQ. The spotted differences indcate the two cases should be modelled separately especially in cases where accuracy is essential.

Table 1: Results obtained for Model 1

Parameters	Independent Characteristics	Dependent characteristics
n*	2	2
ETC	880.93	909.43
PG	0.99982	0.99862
ATI	394	392
AOQ	0.00018	0.000138

5. Conclusion

A model is developed for finding the optimal number of repeat inspection for multi-charactristic critical compoents. An algorithm to find the optimal number of repeat inspections is proposed. An example is presented to demonstrate the developed model and the algorithm. The example is run for both indpendent and dependent failures rates. and a difference in the plan performance measuresis observed. Three measures were used that include average total insection (ATI) and average outgoing quality (AOQ) is observed.

6. References

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